s, depth of heat penetration measured from the phase interfacial surface; δ, liquid overheating on the surface above the boiling point; φ_i, components of the expansion of z in powers of Q^{-1}; $Q=Q_0/cT_к$; $H(1-\xi)$, Heaviside function; $\zeta_j=\zeta(\xi_j)$; $s_j=s(\xi_j)$; $f_j=f(\xi_j)$; ξ_j, instants of time; beginning of melting j = 0, pulse termination j = 1, melting j = 2, crystallization j = c, temperature equilibration in the liquid phase j = 3, $\varepsilon=\varepsilon_m\tau/T_к$; $\gamma=\gamma_m\tau/T_к$; ε_m and γ_m are mean cooling rates of the interphasal and external surfaces.

LITERATURE CITED

1. B. H. Kear, E. M. Breinan, and L. E. Greenwald, Met. Techn., 6, No. 4, 121-129 (1979).
2. S. C. Hsu, S. Chakravorty, and R. Meharabian, Metall. Trans. B. 9, No. 6, 221-229 (1978).
3. D. Ready, Action of Powerful Laser Radiation [Russian translation], Moscow (1974).
4. L. A. Kozdoba, Method of Solving Nonlinear Heat Conduction Problems [in Russian], Moscow (1975).
5. B. Ya. Lyubov, Theory of Crystallization in Large Volumes [in Russian], Moscow (1975).
6. D. Christian, Theory of Transformations in Metals and Alloys [Russian translation] Moscow (1978).
7. L. F. Dona Dalle Rose and A. Miotello, Radiat. Eff. 53, No. 4, 7-18 (1980).
8. A. A. Uglov, I. Yu. Smurov, and A. G. Gus'kov, Fiz. Khim. Obrab. Mater., No. 3, 3-8 (1985).
9. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian] Moscow (1966).
10. V. A. Ditkin and A. P. Prudnikov, Handbook on Operational Calculus [in Russian], Moscow (1965).

A PHYSICAL MODEL FOR SUBLIMATION OF A CONTINUOUS MATERIAL IN AN UNSTABLE DOMAIN

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A physical method is developed for the sublimation process during conductive energy delivery, which permits determination of the configuration and velocity advancement of the phase separation boundaries.

Known methods of analyzing processes associated with the solid—vapor phase transition are based on models in which the position of all points of the interphasal boundary is determined by one coordinate, i.e., the boundary is considered a plane cylindrical of spherical "one-dimensional boundary." However, such an approach often yields results that diverge substantially from the data of practice. An attempt is made in this paper to analyze the sublimation dehydration process during conductive energy supply under conditions when the stability of the one-dimensional sublimation front is spoiled. The instability of a one-dimensional interphasal boundary is understood to be that development of the process for which any fluctuations causing deviation from a one-dimensional surface (lunes, cracks, etc.) that are inevitable under real conditions, increase monotonically. The advancement of the different interphasal surface sections here occurs at a dissimilar velocity because the surface itself acquires a complex configuration. Therefore, necessary for a correct analysis of the process is the determination of the local rate of phase transformation front advancement.

Underlying the physical model is the assumption that the single reason causing a change in phase transition boundary configuration is entrainment of the substance because of its sublimation. Any random distortion of the plane front resulting in diminution of the distance to the heating surface, is accompanied by an increase in the heat flux in the fluctuation zone because of growth of the temperature gradient. In turn, this effect increases the local velocity of the process, which causes a further development of fluctuations. Consequently,

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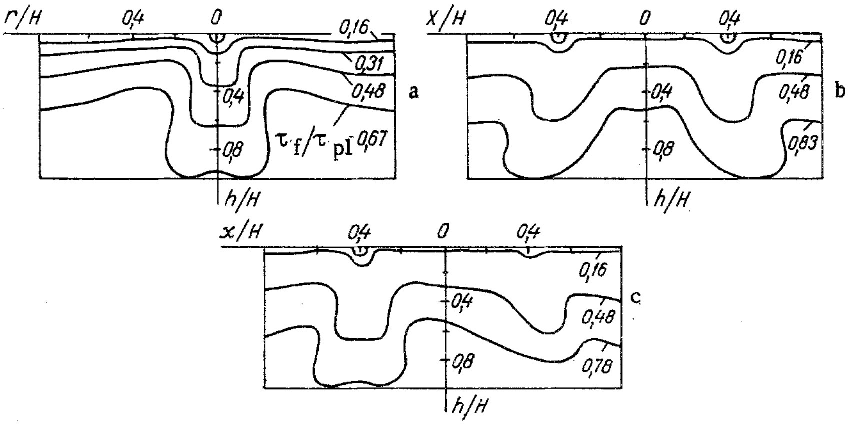

Fig. 1. Development of a non-one-dimensional sublimation front: a) in a disc in the presence of a unit fluctuation; b, c) in a plate in the presence of two fluctuations, symmetric and nonsymmetric cases, respectively.

the initial extension which appears randomly will rapidly be propagated deep into the frozen zone. Moreover, it is assumed that the temperature of the interfacial surface is constant and equal to the equilibrium temperature corresponding to the vapor pressure in the apparatus. In addition, it is considered that the subliming material is isotropic and its thermophysical properties remain unchanged.

Under these assumptions the local rate of advancement of the boundary is determined from the Stefan condition

$$\frac{\partial h}{\partial \tau} = \frac{\lambda}{\rho L} \nabla T. \tag{1}$$

Therefore, the problem reduces to determining the temperature gradient in direct proximity to the boundary, which changes with time in the general case, i.e., to solve the equation

$$\nabla^2 T = \frac{1}{a} \frac{\partial T}{\partial \tau} \tag{2}$$

under the condition (1) on the boundary.

The right side of (2) can be discarded in considering the sublimation of water ice by assuming $\partial T/\partial \tau$ to be small (quasistationary approximation [1]). Additionally assuming that the resistance to the emergence of vapor is negligibly small, we arrive at the Laplace equation for T_p = const.

Because of the complexity of the boundary surface configuration it is not possible to obtain an analytic solution for the development of the unstable front [2], hence, the problem is solved numerically. Cases of the development of a single fluctuation with the shape of a cylinder and a slot cutout as well as two fluctuations occurring simultaneously and with a time shift were investigated.

The stationary temperature field and heat fluxes q_x and q_y were computed on a moving boundary. Then the vertical and horizontal components of the boundary motion velocity were determined. Being given a certain small step in time, a new boundary position was determined and the computation repeated. The results of the computations are represented in Fig. 1 in the form of isotherms.

As the computation showed, the time during which the deepening reaches the heat conducting surface in the presence of a single cylindrical fluctation is 0.67 of the sublimation time under conditions of plane front advancement. For the case of a plane fluctuation in the form of a slot cutout this relationship of the times will equal 0.89. The relative time of the process under conditions of simultaneous development of two fluctuations is 0.83. If the second fluctuation is generated with a lag relative to the first, then because of the interference of both processes the growth of the second extension is slowed down and in the limit

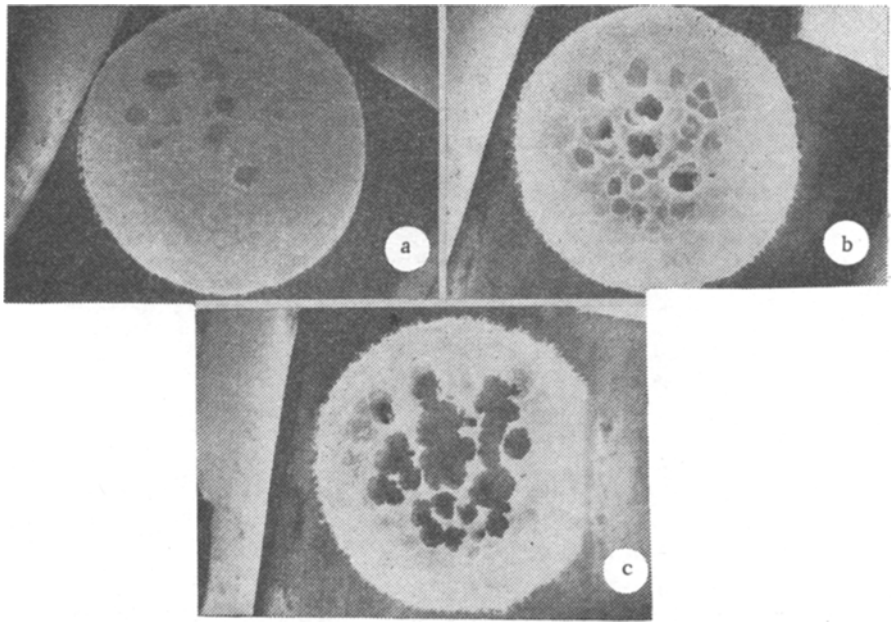

Fig. 2. Development of an unstable sublimation front in an ice monolith under conductive energy supply conditions.

can generally vanish. In the case under consideration the relative time of fluctuation development equals 0.78.

An approximate analytical solution be found for the slotlike inhomogeneity. It was obtained by applying conformal mappings by using the Christoffel—Schwartz integral [3] in the following form

$$\tau_f = \frac{L\rho H^2}{\lambda (T_g - T_v)} \, \frac{(h/H)^2}{2{,}94} \, . \qquad (3)$$

The results of the analytic solution are found in the ratio τ_f/τ_{pl} = 0.68 as compared with the time of plane front advancement.

A series of tests studying the kinetics of sublimation of a continuous material under conductive energy supply was conducted to verify the results obtained. Ice and water—salt solutions frozen in the shape of tablets were investigated. The specimens with thermocouple frozen in them were placed in a vacuum chamber on a substrate heated by a high-amperage current. The temperature in the plane of thermocouple placement was maintained constant in each test by regulation of the delivered power. The propagation time of the sublimated domain to the plain of thermocouple placement was determined by recording the time of thermocouples reading change. Frame-by-frame photorecording of the process was realized in parallel with a 1-sec interval between frames. Characteristic fragments of the process are represented in Fig. 2. The test yields the following value for the magnitude of the relative time τ_f/τ_{pl} = 0.69 ± 0.13.

Most probably the certain deviation of the relative times determined experimentally from the quantity 0.67 computed by a numerical method from the results of a theoretical analysis is a result of experiment error, associated particularly with the complexity of maintaining a constant temperature in the plane of thermocouple placement. However, despite the comparatively large spread in the experimental data (σ = 18.6) the mean value (0.69) differs slightly from that obtained by the numerical method and is in agreement with theory in the 2σ confidence level taken for the experiment.

A deduction can be made on the basis of the results obtained that the duration of the advancement of the non-one-dimensional front can be 20-30% less compared with the time of plane front advancement, however, the total duration of the drying increases here. This is associated with the fact that after passage of the non-one-dimensional front in the layer discrete domains of frozen material remain whose drying rate is ordinarily quite small because of the low heat-conductivity of the dehydrated material. Moreover, the temperature of the heat conducting surface in the case under consideration should be reduced to the level of the temperature of thermal expansion of the product [4], which also results in diminution of the process intensity.

Therefore, questions of the stability of the isometric interfacial phase surfaces should certainly be taken into account in the construction of sublimating installations and the determination of optimal modes for their operation.

NOTATION

λ, heat-conduction coefficient; ρ, density; L, specific heat of sublimation; x, y, h, coordinates; R, H, tablet radius and thickness; q, heat flux; σ, rms deviation; a, thermal diffusivity coefficient; τ_f, time for fluctuation development; τ_{pl}, time of plane front passage; T_r, temperature of the ice surface at the side of the heated substrate; T_e, equilibrium temperature on the sublimation boundary.

LITERATURE CITED

1. A. Z. Volynets, Inzh.-Fiz. Zh., 15, No. 1, 162-164 (1968).
2. Ya. Ber, D. Zaslavski, and S. Irmay, Physicomathematical Principles of Water Filtration [in Russian], Moscow (1971).
3. V. I. Lavrik and V. N. Savenkov, Handbook on Conformal Mappings [in Russian], Kiev (1970).
4. A. M. Brazhnikov, E. I. Kaukhcheshvili, and A. I. Vasil'ev, Technique and Technology of Sublimation Drying of Products, Trudy, KTIRPiKh, No. 69 38-41 (1973).

MELTING OF PORE ICE WITH THE FORMATION OF AN EXTENDED ISOTHERMAL ZONE

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The melting of pore ice, characterized by the formation of a transition zone in which the two phases simultaneously coexist, is investigated. The extent of the transition zone is related to the rate of inflow of water into the melted region of the pore space from outside.

Usually, the melting of ice in the pores of a coarse-grained medium is described mathematically in terms of the classical Stefan problem, on the assumption that the regions of different states of aggregation of the water are separated by a surface of zero thickness. This assumption does not always have to be made and, as shown in [1, 2], even in a homogeneous body it is possible for the front to split and form an extended isothermal zone. Obviously, in composite media, e.g., in frozen sand, in which ice is one of the components, there are more conditions that determine the splitting of the front. One of these is a higher rate of heat transfer in the mineral skeleton than in the pore ice. As a result, there is formed a zone of coexistence of water and ice in which the water coats the warm particles of the skeleton while the ice occupies the centers of the pores.

The solution of the model problem of the melting of sheets of ice alternating with sheets of quartz of equal thickness has shown that this zone is longest at the beginning of the process and subsequently contracts to a small but finite length. This result was obtained on the assumption that no water enters the melting zone from outside, and it is in this case that the formation of a front after a fairly long interval is observed.

The inflow of water from the outside can retard the contraction of the transition zone and ultimately lead to the splitting of the front. The inflow of water is the result of the specific volume of the ice decreasing by an amount $(\rho_w - \rho_i)/\rho_w$, which leads to a sharp decrease in the pressure in the pore space, if it is isolated from the external medium, to a value corresponding to the vapor tension of the water. Later, this observation will be used to estimate the pressure at the leading front.

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